CS325 Artificial Intelligence
Ch. 17.5–6, Game Theory

Cengiz Günay, Emory Univ.

Spring 2013
2012 Competition Complete!

The 2012 StarCraft AIIDE Competition has finished. Click here for unofficial results. The official results will be announced at AIIDE 2012 in October.

Overview

Welcome to the home of the 3rd annual Starcraft AI competition which is organized by the RTS Game AI Research Group at the University of Alberta and sponsored by AIIDE - the AI for Interactive Digital Entertainment conference.

During this event, programs will play Starcraft Broodwar games against each other using BWAPI, a software library that makes it possible to connect programs to the Starcraft: BroodWar game engine.

State of the art subjects: build order planning, over state estimation, plan recognition... Article on 2010 winner: Berkeley Overmind bot
High-level Reinforcement Learning in Strategy Games

Christopher Amato
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Guy Shani
Department of Computer Science
Ben-Gurion University
Beer-Sheva 84105 Israel
shanigu@bgu.ac.il

2010 Paper on playing Civilization IV; uses:

- Markov Decision Processes
- Reinforcement Learning, a model-based Q-learning approach

Compares strategies and parameters on winning outcomes.
And Now, Game Theory

Game theory applies when:

- partially-observable, or
- with simultaneous moves (e.g., StarCraft).
And Now, Game Theory

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- partially-observable, or
- with simultaneous moves (e.g., StarCraft).

> A.I. LOADED >>> ANALYZE LOVE

> A STRANGE GAME. THE ONLY WINNING MOVE IS NOT TO PLAY.
And Now, Game Theory

Game theory applies when:
- partially-observable, or
- with simultaneous moves (e.g., StarCraft).

Game theory deals more with cases like:
- Diplomacy/war between enemies
- Bidding
- Creating win-win scenarios
Exit survey: Adversarial Games

- How do you reduce the tree search complexity of a turn-by-turn game like chess?
- Give an example for a game that we haven’t studied in class which can be solved with the minimax algorithm. Suggest an evaluation function at the cutoff nodes.

Entry survey: Game Theory (0.25 points of final grade)

- Can we use minimax tree search work in simultaneous moves? Briefly explain why or why not?
- Think that you will have to make the move of the US side in a Cold War scenario. How would you consider the opponent’s move, uncertainty, and secrecy?
Terminology: 2 Prisoners Dilemma

More like in Law and Order or The Closer:

- 2 suspects in separate interrogation rooms.

Each can either:

1. Testify against the other, or
2. Refuse to speak.

\[ \begin{align*}
A & : \text{testify} & A & = -5, B & = -5 \\
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A & : \text{refuse} & A & = 0, B & = -10 \\
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Dominant strategy: Selfish decision that is always better.

For A and B?

Testifying is dominant.

Pareto optimal: If no better solution for both players exist.

Which condition?

Three of them.

Nash equilibrium: Local minima; single player switch does not improve.

Is there one?

Testifying, again.
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Console producer \((A)\) vs. game developer \((B)\), need to decide between:

- Blu-ray vs. DVD

Dominant strategy: Selfish decision that is always better.

For \(A\) and \(B\)? None!

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Is there one? Two cases.
Terminology (2): Game Console Game

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Strategies: 2 Finger Morra Game

Difficult, zero-sum betting game:
1. Show a number of fingers
2. Player betting on odd (O) or even (E) wins based on total fingers

Dominant strategy: Selfish decision that is always better.

Utility of E: −3 ≤ U_E ≤ 2

Not very sure? Handicapped? Use mixed strategy

Günay
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![Game Tree Diagram](image)
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- Utility of $E$: $-3 \leq U_E \leq 2$
- Not very sure? Handicapped?
- Use mixed strategy
Calculate $E$'s utility for both players' mixed strategies.

Mixed strategy: Leave opponent no choice!

Parameterize with probabilities:

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$2p - 3(1-p)$ \[ E \]

$-3q + 4(1-q)$
Mixed Strategy: 2 Finger Morra

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Mixed Strategy: 2 Finger Morra

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- Calculate $E$’s utility for both players’ mixed strategies.
- **Mixed strategy:** Leave opponent no choice!

Optimal $p$ for $E$:

\[2p - 3(1 - p) = -3p + 4(1 - p)\]

\[p = \frac{7}{12}\]

\[U_E = 2p - 3(1 - p) = -1/12\]
Mixed Strategy: 2 Finger Morra

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\[ p = \frac{7}{12} \]

\[ U_E = 2p - 3(1 - p) = -\frac{1}{12} \]

Optimal $q$ for $O$:

\[ 2q - 3(1 - q) = -3q + 4(1 - q) \]

\[ q = \frac{7}{12} \]

\[ U_E = 3q + 4(1 - q) = -\frac{1}{12} \]
**Mixed Strategy: 2 Finger Morra**

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<td>$E = +4$</td>
</tr>
</tbody>
</table>

- **Calculate $E$’s utility** for both players’ mixed strategies.
- **Mixed strategy:** Leave opponent no choice!

Optimal $p$ for $E$:

\[
2p - 3(1 - p) = -3p + 4(1 - p) \\
p = \frac{7}{12} \\
U_E = 2p - 3(1 - p) \\
= -\frac{1}{12} \leq U_E \leq
\]

Optimal $q$ for $O$:

\[
2q - 3(1 - q) = -3q + 4(1 - q) \\
q = \frac{7}{12} \\
U_E = 3q + 4(1 - q) \\
= -\frac{1}{12}
\]
Secrecy and Rationality:

Secrecy: If a dominant strategy exists, your opponent can guess it!

Rationality: Sometimes it’s better to look crazy to make your opponent believe you will do something irrational.
Mixed Strategy Issues

Secrecy and Rationality:

**Secrecy:** If a dominant strategy exists, your opponent can guess it!

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A riddle for you.
Mixed Strategy Example

Zero-sum game with min and max:

<table>
<thead>
<tr>
<th></th>
<th>▽: 1</th>
<th>▽: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>△: 1</td>
<td>△= 5</td>
<td>3</td>
</tr>
<tr>
<td>△: 2</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Let’s solve it?

No, need! Dominant strategies exist!

We know what to do

We can guess the other rational player's move

Therefore, $U_E = 5$. 
Mixed Strategy Example

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No, need! **Dominant strategies exist!**

1. We know what to do
2. We can guess the other rational player’s move
Mixed Strategy Example

Zero-sum game with min and max:

\[
\begin{array}{|c|c|}
\hline
\nabla: 1 & \nabla: 2 \\
\hline
\triangle: 1 & \begin{array}{c}
\triangle = 5 \\
4
\end{array} \\
\hline
\triangle: 2 & \begin{array}{c}
3 \\
2
\end{array} \\
\hline
\end{array}
\]

Let’s solve it?
No, need! **Dominant strategies exist!**

1. We know what to do
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Therefore,

\[ U_E = 5. \]
Another Mixed Strategy Example

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</tr>
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<tbody>
<tr>
<td>△: 1</td>
<td>△= 3</td>
<td>6</td>
</tr>
<tr>
<td>△: 2</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Dominant strategies?

\[ p + 5(1-p) = 6 \]
\[ p + 4(1-p) = 4 \]
\[ p = \frac{1}{4} \]

\[ U_△ = 4 \]

Based on △'s decision, \[ U_△ = 3q + 6(1-q) = 6 \] or \[ U_△ = 5q + 4(1-q) = 4 \]

Therefore, \[ 4 \leq U_△ \leq 6 \].
Another Mixed Strategy Example

Dominant strategies? None for $\triangle$.

- Need to calculate only probability $p$, because dominant $q = 0$. 

\[
\begin{array}{c|c|c}
\nabla: 1 & \nabla: 2 \\
\hline
\triangle: 1 & \triangle: 3 & 6 \\
\triangle: 2 & 4 & 5 \\
\end{array}
\]
Another Mixed Strategy Example

<table>
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</tr>
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<tbody>
<tr>
<td>△: 1</td>
<td>△:= 3</td>
</tr>
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<td>4</td>
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Dominant strategies? None for △.

- Need to calculate only probability $p$, because dominant $q = 0$.

\[
3p + 5(1 - p) = 6p + 4(1 - p)
\]

\[
p = \frac{1}{4}
\]

\[
U_\triangle = 4.5
\]
Dominant strategies? None for $\triangle$.

- Need to calculate only probability $p$, because dominant $q = 0$.

Based on $\triangle$’s decision,

$$3p + 5(1 - p) = 6p + 4(1 - p)$$

$$p = \frac{1}{4}$$

$$U_\triangle = 4.5$$
Another Mixed Strategy Example

\[
\begin{array}{c|c|c}
\n & \nabla: 1 & \nabla: 2 \\
\hline
\Delta: 1 & \Delta = 3 & 6 \\
\Delta: 2 & 4 & 5 \\
\end{array}
\]

Dominant strategies? None for $\Delta$.

- Need to calculate only probability $p$, because dominant $q = 0$.

Based on $\Delta$’s decision,

\[
3p + 5(1 - p) = 6p + 4(1 - p)
\]

\[
p = \frac{1}{4}
\]

\[
U_\Delta = 4.5
\]

\[
U_\Delta = 3q + 6(1 - q)
\]

\[
= 6
\]

or

\[
U_\Delta = 5q + 4(1 - q)
\]

\[
= 4
\]

Therefore,

\[
4 \leq U_\Delta \leq 6.
\]
Simplified!
- Deck has only ace and kings: AAKK
- Deal: 1 card each

Rounds:
1. raise/check
2. call/fold

Sequential game/ extensive form
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Sequential game/extensive form

1/6: AA
1/3: AK
1/6: KK
1/3: KA

I1,1
I2,1
I2,2
I2,1

0,0
+1,-1
+2,-2
+1,-1
0,0
+1,-1
-2,+2
+1,-1
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Sequential game/extensive form

\[
\begin{array}{c|c|c|c|c}
 & 1/6: AA & 1/3: AK & 1/6: KK & 1/3: KA \\
\hline
1 & r & 0,0 & +1,-1 & +2,-2 \\
\hline
2 & e & +1,-1 & +1,-1 & +1,-1 \\
\hline
0 & k & -1,+1 & 0,0 & -2,+2 \\
\hline
\end{array}
\]
Poker

Simplified!

- Deck has only ace and kings: AAKK
- Deal: 1 card each

Rounds:
- 1 raise/check
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Sequential game/
extensive form

- Real game has how many states?

\[
\begin{array}{ccc}
1/6: AA & r & 0,0 \\
1/3: AK & k & +1,-1 \\
1/6: KK & k & 0,0 \\
1/3: KA & k & -1,+1 \\
\end{array}
\]
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Sequential game/ extensive form
- Real game has how many states? $\sim 10^{18}$
So How To Solve Non-Simplified Games?

Strategies:

- abstracting; lumping together:
  - Don’t care about aces’ suits, all aces equal
  - Lump similar cards together: cards 1–7 together
  - Bets: small and large
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In summary, game theory is:

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- Not good for: unknown actions, continuous actions, irrational opponents, unknown utility
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Game: Feds vs. Politicians

Game between:
1. Federal reserve and
2. Politicians

on controlling the budget.

Find equilibrium below:
Game: Feds vs. Politicians

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Strategy: design game to have dominant strategy

- Example: second-price auctions (like eBay)